Minimum BER FIR Receiver Filters for DS-CDMA Systems

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Keywords: CDMA, minimum BER, receiver optimization, finite impulse response receiver filter

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I. INTRODUCTION

CDMA is a multiple access technique where the user separation is done neither in frequency, nor in time, but rather through the use of codes. However, the frequency selective fading channel destroys in many cases the codes separation capability and equalization is needed at the receiver. Since the start of the 90's, multi-user detection [1], [2], [3] has provided different multi-user receivers with different performance/complexity trade-offs. Usual target metrics concern either maximizing the likelihood, the spectral efficiency or minimizing the mean square error. In many cases, analytical expressions of the multi-user receivers can be obtained which depend mainly on the noise structure, the impulse channel response, and the nature of the codes.

In the present work, minimum BER is used as a target metric for designing the DS-CDMA receiver filters. Various works, see, e.g., [4], [5], [6], have minimized BER with respect to the receiver parameters (mainly the channel impulse responses in a perfect synchronized system) when the receiver is modeled by a *memoryless* transform.

In this contribution, a general framework based on the discrete-time equivalent low-pass representation of signals is provided. In particular: i) Exact BER expressions are derived for an uplink multi-user DS-CDMA system. ii) The receiver has one FIR multiple-input single-output (MISO) filter for each user, and the significant performance improvements achieved using receiver filters with finite memory are demonstrated. iii) An iterative numerical algorithm is proposed based on the BER expression for finding the complex-valued minimum BER FIR MISO receiver filter coefficients, for given spreading codes and known channel impulse responses. The additive noise on the channel can be colored and complex-valued. iv) Several properties of the minimum BER filters are identified.

II. DS-CDMA MODEL

A. Special Notations

In this article, all the indexing begins with 0. Let $\boldsymbol{A}(z) = \sum_{i=0}^{\eta} \boldsymbol{a}(i) z^{-i}$ be an FIR MIMO filter of order η and size $M_0 \times M_1$. The matrix $\boldsymbol{a}(i)$ is the ith coefficient of the FIR MIMO filter $\boldsymbol{A}(z)$ and it has size $M_0 \times M_1$. The *row-expanded* matrix $\boldsymbol{A}_{\_}$ obtained from the FIR MIMO filter $\boldsymbol{A}(z)$ is an $M_0 \times (\eta+1)M_1$ matrix given by: $\boldsymbol{A}_{\_} = [\boldsymbol{a}(0) \;\; \boldsymbol{a}(1) \;\; \cdots \;\; \boldsymbol{a}(\eta)]$. Let q be a non-negative integer. The *row-diagonal-expanded* matrix $\boldsymbol{A}_{\urcorner}^{(q)}$ of the FIR MIMO filter $\boldsymbol{A}(z)$ of order q is a $(q+1)M_0 \times (\eta+q+1)M_1$ block Toeplitz matrix given by:

$$\boldsymbol{A}_{\urcorner}^{(q)} = \begin{bmatrix} \boldsymbol{a}(0) & \cdots & \boldsymbol{a}(\eta) & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{a}(0) & \cdots & \cdots & \boldsymbol{a}(\eta) \end{bmatrix}. \qquad (1)$$

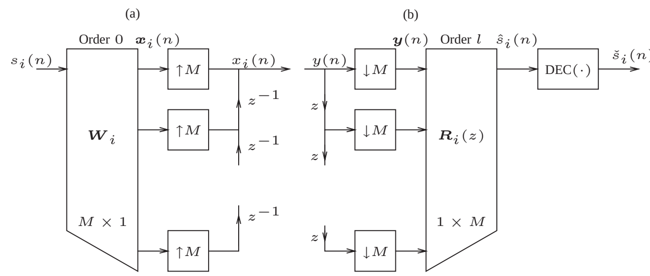

Fig. 1. (a) DS-CDMA transmitter number i, and (b) DS-CDMA receiver part designed for decoding user number i.

Let ν be a non-negative integer. The symbol n is used as a time index in this article and n is an integer. Let $\boldsymbol{y}(n)$ be a vector time-series of size $M \times 1$. The column expansion of $\boldsymbol{y}(n)$ of order ν has size $(\nu+1)M \times 1$ and is defined as: $\boldsymbol{y}(n)_{|}^{(\nu)} = \left[\boldsymbol{y}^T(n), \boldsymbol{y}^T(n-1), \cdots, \boldsymbol{y}^T(n-\nu)\right]^T$.

B. System Experienced by User Number i

It is assumed that the input $s_i(n)$ sent by user number $i \in \{0, 1, \ldots, N-1\}$ is an independent and identically distributed time-series, uncorrelated with the additive channel noise and the data sequences sent by the other users. The transmitted sequences $s_i(n)$ are BPSK modulated signals, such that $s_i(n) \in \{-1, +1\}$, with equally likely symbols. The information symbols $s_i(n)$ are spread with a spreading code having spreading factor M. Let the vector $\boldsymbol{W}_i$ be an $M \times 1$ vector containing the spreading code for user number i. The vector $\boldsymbol{W}_i$ is an FIR single-input multiple-output (SIMO) filter with zero order that increases the sampling rate of the original signal by the factor M. It is assumed that the receiver knows the values of all the vectors $\boldsymbol{W}_i$, and they can be chosen arbitrarily, i.e., $\boldsymbol{W}_i \in \mathbb{C}^{M \times 1}$, where $\mathbb{C}$ denotes the set of complex number. Figure 1 (a) shows the ith transmitter of the DS-CDMA system and the DS-CDMA receiver part that is designed to decode user number i is shown by Figure 1 (b). In Figure 1, z^{-1} is the delay element, z is the advance element, $\uparrow M$ is expansion with factor M meaning that $M-1$ zeros are inserted between each sample, and $\downarrow M$ is decimation by M. All these four building blocks are standard elements in digital signal processing [7]. In order to produce the $M \times 1$ vector $\boldsymbol{y}(n)$ from a scalar time-series $y(n)$, see Figure 1 (b), the following blocking structure is used $\boldsymbol{y}(n) = [y(nM), y(nM+1), \cdots, y(nM+M-1)]^T$, where the operator $(\cdot)^T$ denotes transposition. The input sequence $x_i(n)$ to the ith channel, see Figure 1 (a), is stacked into a $M \times 1$ vector $\boldsymbol{x}_i(n)$ according to $\boldsymbol{x}_i(n) = [x_i(Mn), x_i(Mn+1), \cdots, x_i(Mn+M-1)]^T$. The spreading operation may be written as $\boldsymbol{x}_i(n) = \boldsymbol{W}_i s_i(n)$. Let p be a non-negative integer. Using the previous notations, the $(p+1)M \times 1$ vector $\boldsymbol{x}_i(n)_{|}^{(p)}$ can be expressed as $\boldsymbol{x}_i(n)_{|}^{(p)} = \boldsymbol{W}_{i\,\urcorner}^{(p)} \boldsymbol{s}_i(n)_{|}^{(p)}$, where $\boldsymbol{W}_{i\,\urcorner}^{(p)} = \boldsymbol{I}_{p+1} \otimes \boldsymbol{W}_i$ has size $(p+1)M \times (p+1)$, where $\otimes$ is the Kronecker product, and $\boldsymbol{s}_i(n)_{|}^{(p)} = [s_i(n), s_i(n-1), \cdots, s_i(n-p)]^T$ has size $(p+1) \times 1$.

The ith user has the following scalar multipath channel transfer function: $C_i(z) = \sum_{k=0}^{L} c_i(k) z^{-k}$. The maximum order of all N channels is L. It is assumed that $L \leq M$. When $L \leq M$, it is shown in [8], that the equivalent FIR MIMO channel filter $\boldsymbol{C}_i(z)$ of size $M \times M$ has order $q = 1$, when the blocking and unblocking operations in Figure 1 are used. $\boldsymbol{C}_i(z)$ is given by: $\boldsymbol{C}_i(z) = \boldsymbol{c}_i(0) + \boldsymbol{c}_i(1) z^{-1}$, where the two matrix channel coefficients are given by

$$\boldsymbol{c}_i(0) = \begin{pmatrix} c_i(0) & 0 & 0 & \cdots & 0 \\ \vdots & c_i(0) & 0 & \cdots & 0 \\ c_i(L) & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \ldots & c_i(L) & \ldots & c_i(0) \end{pmatrix},$$

$$\boldsymbol{c}_i(1) = \begin{pmatrix} 0 & \cdots & c_i(L) & \cdots & c_i(1) \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & c_i(L) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{pmatrix}. \tag{2}$$

The channel is assumed to be corrupted by zero-mean additive Gaussian complex circularly symmetric noise, denoted $v(n)$, which is independent of the transmitted signals. The additive channel noise vector $\boldsymbol{v}(n)$ of size $M \times 1$ can be expressed as: $\boldsymbol{v}(n) = [v(Mn), v(Mn+1), \cdots, v(Mn+M-1)]^T$. The channel noise is assumed to have known second-order statistics, which might be colored in general. The autocorrelation matrix of size $(l+1)M \times (l+1)M$ of the $(l+1)M \times 1$ vector $\boldsymbol{v}(n)_{|}^{(l)}$ is defined as $\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)} = E\left[\boldsymbol{v}(n)_{|}^{(l)} \left(\boldsymbol{v}(n)_{|}^{(l)}\right)^H\right]$, where the operator $(\cdot)^H$ denotes complex conjugated transposed. Let the variance of the components of the complex Gaussian circularly symmetric additive channel noise $\boldsymbol{v}(n)$ be given by $N_0 = 1/M \operatorname{Tr}\left\{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(0,M)}\right\}$, where $\operatorname{Tr}\{\cdot\}$ is the trace operator. The average energy per bit E_b at the input of the channels is given by $E_b = 1/N \sum_{i=0}^{N-1} E\left[\boldsymbol{x}_i^H(n)\boldsymbol{x}_i(n)\right] = 1/N \sum_{i=0}^{N-1} \boldsymbol{W}_i^H \boldsymbol{W}_i$. Let the channel condition be defined as the value of the energy per bit to noise ratio, i.e., E_b/N_0.

The desired signal at the output of the receiver filter number i is $d_i(n) = s_i(n-\delta)$, where $\delta \in \{0, 1, \ldots, l+1\}$ denotes the decision delay and δ is equal for all N users. Receiver filter number i takes the $M \times 1$ input vector $\boldsymbol{y}(n)$ and produces a scalar as its output, see Figure 1 (b). The size of the ith receiver filter is $1 \times M$, and its transfer function $\boldsymbol{R}_i(z)$ is given by

$$\boldsymbol{R}_i(z) = \sum_{k=0}^{l} \boldsymbol{r}_i(k) z^{-k}, \tag{3}$$

where $\boldsymbol{r}_i(k)$, of size $1 \times M$, is filter coefficient number k of receiver filter number i. The order l is assumed to be fixed and known. Since uplink is considered, the receiver is trying to estimate the transmitted bits from all of the N users by means of N FIR MISO receiver filters. At the output of the FIR MISO receiver filter $\boldsymbol{R}_i(z)$, a decision device is used to recover the original data information bits. The blocks denoted $\mathrm{DEC}(\cdot)$ estimate the output bits $\check{s}_i(n)$ by taking the real value of a complex-valued sequence $\hat{s}_i(n)$ and then a hard decision is made returning $+1$ if $\hat{s}_i(n)$ is non-negative and -1 if $\hat{s}_i(n)$ is negative. The used memoryless decisions units are suboptimal and better performance can be obtained if more advanced soft decoding techniques are employed.

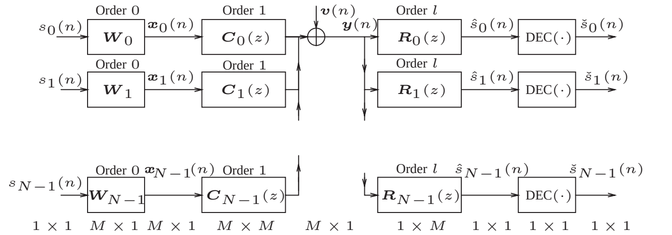

Fig. 2. Block model of the N users DS-CDMA system.

C. Block Description

A block description of the DS-CDMA system is shown in Figure 2. All the input signals of the system are assumed to be jointly wide sense stationary (WSS).

D. Input-Output Relationship

The row-expanded FIR filter, of size $1 \times (l+2)$, from the input of transmitter number i to the output of the receiver filter number i is given by $\boldsymbol{R}_{i\_}\boldsymbol{C}_{i\urcorner}^{(l)}\boldsymbol{W}_{i\urcorner}^{(l+1)}$. The received signal vector $\boldsymbol{y}(n)$ can be expressed as

$$\boldsymbol{y}(n) = \sum_{i=0}^{N-1} \boldsymbol{C}_{i\_}\boldsymbol{W}_{i\urcorner}^{(1)} \boldsymbol{s}_i(n)_{|}^{(1)} + \boldsymbol{v}(n), \tag{4}$$

where the vector $\boldsymbol{s}_i(n)_{|}^{(l+1)}$ has size $(l+2) \times 1$.

Let the vector $\boldsymbol{s}_i(n)$ have size $(l+2)N \times 1$ and be defined as: $\boldsymbol{s}(n) = \left[\left(\boldsymbol{s}_0(n)_{|}^{(l+1)}\right)^T, \left(\boldsymbol{s}_1(n)_{|}^{(l+1)}\right)^T, \cdots, \left(\boldsymbol{s}_{N-1}(n)_{|}^{(l+1)}\right)^T\right]^T$. The $(l+2)N \times 1$ vector $\boldsymbol{s}^{(i)}(n)$ is defined as: $\boldsymbol{s}^{(i)}(n) = \boldsymbol{s}(n)\left(\boldsymbol{s}(n)\right)_{(l+2)i+\delta} = \boldsymbol{s}(n) s_i(n-\delta)$.

The convolution of the zero order FIR SIMO filter $\boldsymbol{W}_k$ and the first order FIR MIMO channel transfer function $\boldsymbol{C}_k(z)$ is denoted by $\boldsymbol{B}_k(z)$, and $\boldsymbol{B}_k(z)$ has size $M \times 1$, and order 1. The row expansion of $\boldsymbol{B}_k(z)$ is given by $\boldsymbol{B}_{k\_} = \boldsymbol{C}_{k\_}\boldsymbol{W}_{k\urcorner}^{(1)}$, and $\boldsymbol{B}_{k\_}$, $\boldsymbol{C}_{k\_}$, and $\boldsymbol{W}_{k\urcorner}^{(1)}$ have size $M \times 2$, $M \times 2M$, and $2M \times 2$, respectively. The row-diagonal expansion of $\boldsymbol{B}_k(z)$ of order l is given by $\boldsymbol{B}_{k\urcorner}^{(l)} = \boldsymbol{C}_{k\urcorner}^{(l)}\boldsymbol{W}_{k\urcorner}^{(l+1)}$, and $\boldsymbol{B}_{k\urcorner}^{(l)}$, $\boldsymbol{C}_{k\urcorner}^{(l)}$, and $\boldsymbol{W}_{k\urcorner}^{(l+1)}$ have size $(l+1)M \times (l+2)$, $(l+1)M \times (l+2)M$, and $(l+2)M \times (l+2)$, respectively. Let the matrix $\boldsymbol{T}$ be defined as: $\boldsymbol{T} = \left[\boldsymbol{B}_{0\urcorner}^{(l)}, \boldsymbol{B}_{1\urcorner}^{(l)}, \cdots, \boldsymbol{B}_{N-1\urcorner}^{(l)}\right]$. and it has size $(l+1)M \times (l+2)N$.

The output of the ith receiver filter at time instance n is denoted by $\hat{s}_i(n)$ and it is given by $\hat{s}_i(n) = \boldsymbol{R}_{i\_}\boldsymbol{y}(n)_{|}^{(l)}$. It follows from (4), that $\boldsymbol{y}(n)_{|}^{(l)}$ is given by: $\boldsymbol{y}(n)_{|}^{(l)} = \sum_{k=0}^{N-1} \boldsymbol{C}_{k\urcorner}^{(l)}\boldsymbol{W}_{k\urcorner}^{(l+1)}\boldsymbol{s}_k(n)_{|}^{(l+1)} + \boldsymbol{v}(n)_{|}^{(l)}$. The overall expression for the output signal of receiver filter number i can be written as

$$\hat{s}_i(n) = \boldsymbol{R}_{i\_}\boldsymbol{T}\boldsymbol{s}(n) + \boldsymbol{R}_{i\_}\boldsymbol{v}(n)_{|}^{(l)}. \tag{5}$$

E. Minimum MSE Receiver

The average MSE over all the N users is defined as $\mathrm{MSE} = 1/N \sum_{i=0}^{N-1} \mathrm{MSE}_i$, where MSE_i is the MSE of the ith user: $\mathrm{MSE}_i = E\left[|\hat{s}_i(n) - d_i(n)|^2\right]$. It can be shown that MSE_i is given by

$$\begin{aligned} \mathrm{MSE}_i = {} & \boldsymbol{R}_{i\_}\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\left(\boldsymbol{R}_{i\_}\right)^H + 1 - \boldsymbol{R}_{i\_}\boldsymbol{T}\boldsymbol{e}_{(l+2)i+\delta} \\ & - \left(\boldsymbol{e}_{(l+2)i+\delta}\right)^H \boldsymbol{T}^H \left(\boldsymbol{R}_{i\_}\right)^H + \boldsymbol{R}_{i\_}\boldsymbol{T}\boldsymbol{T}^H\left(\boldsymbol{R}_{i\_}\right)^H, \end{aligned} \tag{6}$$

where $\boldsymbol{e}_k$ is the unit vector of size $(l+2)N \times 1$ with $+1$ in position number k and zeros elsewhere. By using derivation with respect to $\boldsymbol{R}_{i\_}^*$, where * means complex conjugation, the minimum MSE receiver filter number i is given by:

$$\boldsymbol{R}_{i\_} = \left(\boldsymbol{e}_{(l+2)i+\delta}\right)^T \boldsymbol{T}^H \left[\boldsymbol{T}\boldsymbol{T}^H + \boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right]^{-1}. \tag{7}$$

F. Noise-Free Eye Diagrams

There exist $2^{N(l+2)}$ different realizations for the vector $\boldsymbol{s}(n)$. Let $\boldsymbol{s}_k(n)$ be one of these vectors $\boldsymbol{s}(n)$, where $k \in \{0, 1, \ldots, 2^{N(l+2)} - 1\}$, and define the $(l+2)N \times 1$ vector $\boldsymbol{s}_k^{(i)}(n)$ as $\boldsymbol{s}_k^{(i)}(n) = \boldsymbol{s}_k(n)\left(\boldsymbol{s}_k(n)\right)_{(l+2)i+\delta}$, where the operator $(\cdot)_k$ denotes component number k of the vector it is applied to. Whenever the index k is not required, $\boldsymbol{s}^{(i)}(n)$ might be used to denote one of the $\boldsymbol{s}_k^{(i)}(n)$ vectors. Since $(\boldsymbol{s}_k(n))_k \in \{-1, 1\}$, the vector $\boldsymbol{s}^{(i)}(n)$ will always contain $+1$ in the vector component number $(l+2)i + \delta$. Therefore, there exists a total of

$$K \triangleq 2^{(l+2)N-1}, \tag{8}$$

different $\boldsymbol{s}^{(i)}(n)$ vectors. Let the symbol $\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}$ denoting the kth vector of size $(l+1)M \times 1$, be defined as: $\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)} \triangleq \boldsymbol{T}\boldsymbol{s}_k^{(i)}(n)$. As seen from the right-hand side of (5), $\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}$ is the column vector expansion of order l of the noise-free input vector to the receiver, of size $(l+1)M \times 1$, when the vector $\boldsymbol{s}_k^{(i)}(n)$ was sent from the transmitters. If the indexing of k is not needed, the symbol $\boldsymbol{t}^{(i)}(n)_{|}^{(l)} = \boldsymbol{T}\boldsymbol{s}^{(i)}(n)$ can be used. The vector $\left(\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right)^H \left[\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right]^{-1}$ has size $1 \times (l+1)M$, and this vector is named the *receiver-signal vector*.

Let the operator $\mathrm{Re}\{\cdot\}$ denote the real part of the scalar it is applied to. It is assumed that the system is synchronized such that the noise-free eye diagrams here is in the middle of their analogue counterparts. The positive part of the ith noise-free eye diagram at time instant n is defined as the real part of the noise-free signal at the output of the receiver filter $\boldsymbol{R}_i(z)$ at time n when the desired signal is $d_i(n) = s_i(n-\delta) = +1$. From (5) and Figure 2, it can be seen that $\mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{T}\boldsymbol{s}^{(i)}(n)\right\}$ is the real part of the output of the ith FIR MISO receiver filter $\boldsymbol{R}_i(z)$ at time n when the vector given by $\boldsymbol{s}^{(i)}(n)$ was transmitted with no channel noise. At time n, the ith FIR receiver filter $\boldsymbol{R}_{i\_}$ is trying to estimate the value of the desired signal $d_i(n) = s_i(n-\delta)$. In the vector $\boldsymbol{s}^{(i)}(n)$, the value corresponding to $s_i(n-\delta)$ is equal to $+1$ due to the definition of the vector $\boldsymbol{s}^{(i)}(n)$. The positive part of the ith noise-free eye diagram can be expressed as

$$\mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{T}\boldsymbol{s}_k^{(i)}(n)\right\} = \mathrm{Re}\left\{\left\langle \boldsymbol{R}_{i\_}, \left(\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right)^H \left[\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right]^{-1} \right\rangle_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}\right\}, \tag{9}$$

where $i \in \{0, 1, \ldots, N-1\}$ and $k \in \{0, 1, \ldots, K-1\}$. The *receiver inner product* $\langle \boldsymbol{f}_0, \boldsymbol{f}_1 \rangle_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}} \triangleq \boldsymbol{f}_0 \boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)} \boldsymbol{f}_1^H$ is used here. If the system has an open noise-free eye diagram at the output of the ith receiver filter, then the expressions in (9) must be positive for all $k \in \{0, 1, \ldots, K-1\}$.

G. Definitions

Definition 1: Let user number i have spreading code of length M given by $\boldsymbol{W}_i$ and let the $M \times M$ channel block transfer matrices $\boldsymbol{C}_i(z)$ be given. These channels are said to be (l, δ) *linear FIR equalizable* if there exists N linear FIR MISO receiver filters $\boldsymbol{R}_i(z)$ with size $1 \times M$ and order l, see (3), such that all the N noise-free eye diagrams are open when the delay through the system is δ.

Remark 1: Note that there exist channels that are not linear FIR equalizable for $(l, \delta) = (0, 0)$, but the same channels might be linear FIR equalizable for larger values of l or δ. There exist *scalar channels* that are not linear FIR equalizable for some values of N and M, but if these values are sufficiently increased, then the communication system becomes linear FIR equalizable.

Definition 2: The ith *receiver-signal set* $\mathcal{R}_i$ is defined as:

$$\mathcal{R}_i = \left\{ \sum_{k=0}^{K-1} g_k \left(\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right)^H \left[\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right]^{-1} \middle| \, g_k > 0 \right\}. \tag{10}$$

For linear FIR equalizable channels, it is seen from the equality in (9) that there exists at least one receiver $\boldsymbol{R}_{i\_}$ that has a positive real part of the receiver inner product with all the receiver-signal vectors. Since the receiver-signal vectors generate the set $\mathcal{R}_i$, see (10), the set $\mathcal{R}_i$ is a cone when the channels are linear FIR equalizable. The set in (10) are called receiver-cone, when the channels are linear FIR equalizable.

In general, for linear FIR equalizable channels, only *subsets* of the receiver-signal cones will result in open noise-free eye diagrams. From (9), it is seen that for linear FIR equalizable channels, the ith noise-free eye diagram is open if the following condition is satisfied: The vector $\boldsymbol{R}_{i\_}$ lies inside the *subset* of $\mathcal{R}_i$ that has a positive real part of receiver inner product with all the receiver-signal vectors.

H. Exact Expression of the BER

The total average BER for the system given in Figure 2 can be expressed as:

$$\mathrm{BER} = \frac{1}{N} \sum_{i=0}^{N-1} \mathrm{BER}_i \,. \tag{11}$$

BER_i is the BER of vector component number i of the output vector $\check{\boldsymbol{s}}(n)$, and it can be expressed as

$$\begin{aligned}
&\mathrm{BER}_i = \Pr\{\check{s}_i(n) \neq s_i(n-\delta)\} = \Pr\{\mathrm{Re}\{\hat{s}_i(n)\}\, s_i(n-\delta) < 0\} \\
&= \Pr\left\{\mathrm{Re}\left\{\boldsymbol{R}_{i_}\boldsymbol{T}\boldsymbol{s}(n) + \boldsymbol{R}_{i_}\boldsymbol{v}(n)_{|}^{(l)}\right\} s_i(n-\delta) < 0\right\} \\
&= \Pr\left\{\mathrm{Re}\left\{\boldsymbol{R}_{i_}\boldsymbol{T}\boldsymbol{s}^{(i)}(n) + \boldsymbol{R}_{i_}\boldsymbol{v}(n)_{|}^{(l)} s_i(n-\delta)\right\} < 0\right\} \\
&= \Pr\left\{-\mathrm{Re}\left\{\boldsymbol{R}_{i_}\boldsymbol{v}(n)_{|}^{(l)} s_i(n-\delta)\right\} > \mathrm{Re}\left\{\boldsymbol{R}_{i_}\boldsymbol{t}^{(i)}(n)_{|}^{(l)}\right\}\right\} \\
&= E\Big[\Pr\Big\{-\mathrm{Re}\left\{\boldsymbol{R}_{i_}\boldsymbol{v}(n)_{|}^{(l)} s_i(n-\delta)\right\} \\
&\quad > \mathrm{Re}\left\{\boldsymbol{R}_{i_}\boldsymbol{t}^{(i)}(n)_{|}^{(l)}\right\} \Big| \, \boldsymbol{s}(n)\Big\}\Big]\,,
\end{aligned} \tag{12}$$

where $\Pr\{\cdot\}$ is the probability operator and $\Pr\{A\} = E[\Pr\{A|B\}]$ with the expected value taken with respect to B. In (12), $s_i(n-\delta) = (\boldsymbol{s}(n))_{(l+2)i+\delta}$ and the definition of $\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}$ were used. In order to simplify further the expression above, it is important to realize that the left hand side of the last inequality is a real Gaussian stochastic variable with mean and variance

$$E\Big[-\mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{v}(n)_{|}^{(l)} s_i(n-\delta)\right\}\Big] = 0, \tag{13}$$

$$E\Big[\mathrm{Re}^2\left\{\boldsymbol{R}_{i\_}\boldsymbol{v}(n)_{|}^{(l)} s_i(n-\delta)\right\}\Big] = \frac{1}{2}\left\|\boldsymbol{R}_{i\_}\right\|^2_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}\,, \tag{14}$$

where $\mathrm{Re}^2\{\cdot\}$ denotes the squared value of the real part of the argument. By utilizing the distribution of the vectors $\boldsymbol{s}(n)$ and $\boldsymbol{s}_k^{(i)}(n)$, the definition of the Q-function together with the results from (13) and (14), it can be shown that (12) can be rewritten as

$$\begin{aligned}
\mathrm{BER}_i &= E\left[Q\left(\frac{\sqrt{2}\,\mathrm{Re}\left\{\boldsymbol{R}_{i_}\boldsymbol{t}^{(i)}(n)_{|}^{(l)}\right\}}{\left\|\boldsymbol{R}_{i_}\right\|_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}}\right)\right] \\
&= \frac{1}{K}\sum_{k=0}^{K-1} Q\left(\frac{\sqrt{2}\,\mathrm{Re}\left\{\left\langle \boldsymbol{R}_{i_}, \left(\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right)^H \left[\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right]^{-1}\right\rangle_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}\right\}}{\left\|\boldsymbol{R}_{i_}\right\|_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}}\right),
\end{aligned} \tag{15}$$

where (9) was used, and where K is given by (8). The expression for BER is an extension of (3) in [5] to include complex variables and for the case where $l > 0$. For $l = 0$, the expression is also in accordance with (20) in [9], although the expression in [9] contains twice as many terms for each sum over k. The reason is that in [9], it has not been considered that the vectors $\boldsymbol{s}_k^{(i)}(n)$ contain $+1$ in vector component number $(l{+}2)i{+}\delta$, independently of k. Experiments show that there is an excellent match between the theoretical performance given in (11) and performance achieved by Monte Carlo simulations.

I. Receiver Filter Normalization and Problem Formulation

From (11) and (15), it can be deduced that the exact value of the BER is independent of the receiver inner product norm of the vectors $\boldsymbol{R}_{i\_}$ Therefore, there is no loss of optimality by choosing

$$\|\boldsymbol{R}_{i\_}\|^2_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}} = \boldsymbol{R}_{i\_}\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\boldsymbol{R}_{i\_}^{\,H} = 1. \quad (16)$$

The robust receiver design problem can be formulated as:

Problem 1: $\min\limits_{\{\boldsymbol{R}_0(z),\boldsymbol{R}_1(z),\cdots,\boldsymbol{R}_{N-1}(z)\}} \mathrm{BER}.$

III. Minimum BER FIR Receiver Design

A. Properties of the Minimum BER Receiver Filters

Lemma 1: If the channels are linear FIR equalizable, then the minimum BER ith receiver $\boldsymbol{R}_{i\_}$ lies in $\mathcal{R}_i$.

Proof: See [10] ■

Proposition 1: If $\mathrm{BER} < \frac{1}{2K}$, then all the N noise-free eye diagrams are open.

Proof: See [10]. ■

Proposition 2: Assume that the channels are linear FIR equalizable. If the receiver FIR MISO filters are constrained to belong to the sets that have open noise-free eye diagrams and each of the receiver filters $\boldsymbol{R}_{i\_}$ satisfies (20), then the optimized receiver is a global minimum.

Proof: See [10]. ■

B. Numerical Optimization Algorithm

The necessary conditions for optimality of the ith receiver filter can be expressed as: $\frac{\partial}{\partial \boldsymbol{R}_{i\_}^*}\mathrm{BER} = \boldsymbol{0}_{1\times(l+1)M}$. The following two conjugate derivatives will be useful

$$\frac{\partial}{\partial \boldsymbol{R}_{i\_}^*}\mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right\} = \frac{1}{2}\left(\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right)^H, \quad (17)$$

$$\frac{\partial}{\partial \boldsymbol{R}_{i\_}^*}\frac{1}{\|\boldsymbol{R}_{i\_}\|_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}} = \frac{-1}{2\,\|\boldsymbol{R}_{i\_}\|^3_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}}\boldsymbol{R}_{i\_}\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}. \quad (18)$$

By means of (11), (15), and the definition of the Q-function, the necessary conditions for optimality can be reformulated as:

$$\sum_{k=0}^{K-1} e^{-\frac{\mathrm{Re}^2\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right\}}{\|\boldsymbol{R}_{i\_}\|^2_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}}} \Bigg\{\mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right\}\frac{\partial}{\partial \boldsymbol{R}_{i\_}^*}\|\boldsymbol{R}_{i\_}\|^{-1}_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}$$

$$+\frac{1}{\|\boldsymbol{R}_{i\_}\|_{\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}}}\frac{\partial}{\partial \boldsymbol{R}_{i\_}^*}\mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right\}\Bigg\} = \boldsymbol{0}_{1\times(l+1)M}. \quad (19)$$

By introducing the results from (17) and (18) into (19) and using the normalization in (16), then (19) can be rewritten as:

$$\boldsymbol{R}_{i\_} = \frac{\sum\limits_{k_1=0}^{K-1} e^{-\mathrm{Re}^2\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_{k_1}^{(i)}(n)_{|}^{(l)}\right\}}\left(\boldsymbol{t}_{k_1}^{(i)}(n)_{|}^{(l)}\right)^H\left[\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right]^{-1}}{\sum\limits_{k_0=0}^{K-1} e^{-\mathrm{Re}^2\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_{k_0}^{(i)}(n)_{|}^{(l)}\right\}}\mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_{k_0}^{(i)}(n)_{|}^{(l)}\right\}}. \quad (20)$$

The following result now follows immediately.

Theorem 1: Assume that the channels are linear FIR equalizable and that the normalization in (16) is used, then the optimal receiver filter number i satisfies (20) and it lies in $\mathcal{R}_i$.

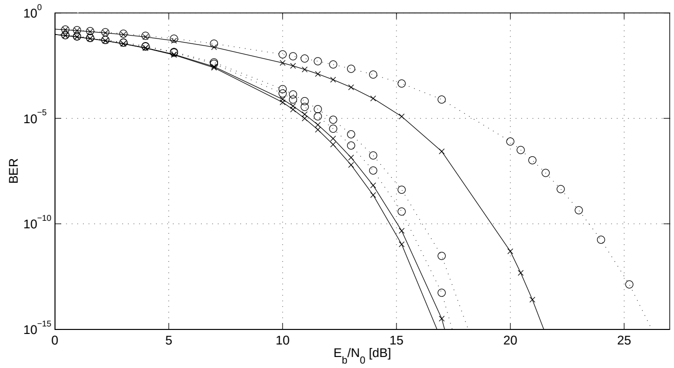

Fig. 3. BER versus E_b/N_0 performances of the minimum MSE DS-CDMA system $(\cdots\circ\cdots)$ and the proposed minimum BER DS-CDMA system $(-\times-)$ for different values of receiver filter order $l \in \{0,1,2\}$, when $M = 7$, $L = 5$, and $N = 3$. When l increases, then the performance curves move downward.

(20) reduces to (12) in [11] when $N = M = 1$, the matrix $\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}$ is proportional to the identity matrix, and only real filters and signals are present.

The steepest decent method is used in the optimization of the ith FIR receiver filter. In [10], it is shown that the following result holds

$$\frac{\partial}{\partial \boldsymbol{R}_{i\_}^*}\mathrm{BER} = \frac{-1}{2\sqrt{\pi}KN}\sum_{k=0}^{K-1} e^{-\mathrm{Re}^2\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right\}}$$

$$\times\left\{\left(\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right)^H - \mathrm{Re}\left\{\boldsymbol{R}_{i\_}\boldsymbol{t}_k^{(i)}(n)_{|}^{(l)}\right\}\boldsymbol{R}_{i\_}\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right\}, \quad (21)$$

when the normalization in (16) is used. The steepest descent method is used in the CDMA receiver optimization. The whole system can be optimized for the different possible values of the delay δ. The initial value for the FIR MISO receiver filter coefficients should be chosen appropriately. One possibility is to use filter coefficients from filters of the same order, where the filters are optimized according to the minimum MSE criterion, see Subsection II-E. When the minimum BER receiver FIR MISO filters have been found for a certain channel condition E_b/N_0, these values can be used as initial values for other channel conditions which are close to the one already optimized. The algorithm is guaranteed to converge at least to a local minimum.

C. Low E_b/N_0 Regime

In [10], it is shown that the minimum BER receiver filters will approach the following result as $E_b/N_0 \rightarrow 0^+$:

$$\boldsymbol{R}_{i\_} = \beta\left(\boldsymbol{e}_{(l+2)i+\delta}\right)^T\boldsymbol{T}^H\left[\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l,M)}\right]^{-1}, \quad (22)$$

where β is a positive constant chosen such that (16) is satisfied. The result in (22) is an extension to the FIR MISO case of the *average matched receiver filter* that is found in [12]. From (22), it follows that the optimal receiver filter number i for bad channel conditions lies in the ith receiver-signal set $\mathcal{R}_i$ given by (10). If the channel noise is very high, it is seen from (7), that minimum MSE receiver number i is proportional to the result in (22).

IV. Results and Comparisons

Here, comparisons are made against the minimum MSE receiver filters given in Subsection II-E, which are also called Wiener filters.

Let the $(L+1)\times 1$ vector $\boldsymbol{h}_i \triangleq [c_i(0), c_i(1), \cdots, c_i(L)]^T$, where $\boldsymbol{h}_i^H\boldsymbol{h}_i = 1$. The channel impulse response coefficients $c_i(k)$ were taken from a white complex Gaussian random process. Real normalized Gold codes [13] were used as spreading codes $\boldsymbol{W}_i$, the delay was chosen as $\delta = \left\lfloor\frac{l+1}{2}\right\rfloor$, and $v(n)$ is white.

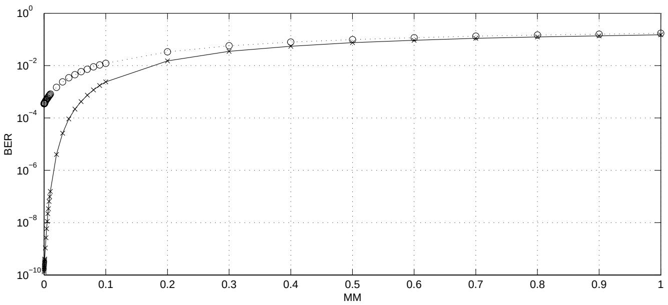

Fig. 4. BER versus MM performances of the minimum MSE DS-CDMA system $(\cdots\circ\cdots)$ and the proposed DS-CDMA system $(-\times-)$, when $l=0$, $M=7$, $L=5$, $N=5$, and $E_b/N_0=20$ dB in all cases.

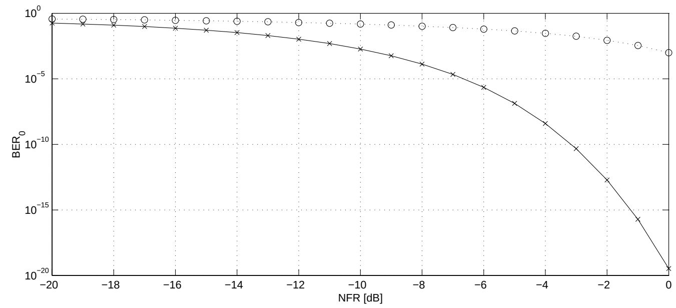

Fig. 5. BER_0 versus NFR performances of the minimum MSE DS-CDMA system $(\cdots\circ\cdots)$ and the proposed minimum BER DS-CDMA system $(-\times-)$, when $l=0$, $M=7$, $L=5$, $N=5$, and $E_b/N_0=20$ dB in all cases.

Figure 3 shows the BER versus E_b/N_0 performances of the minimum MSE and the proposed minimum BER systems when $M=7$, $L=5$, and $N=3$ and $l\in\{0,1,2\}$. When l increases, the performance of the two systems improves. From Figure 3, it is seen that a significant improvement can be achieved by increasing l from 0 to 1 in this example, however, there is only a small improvement in performance when l increases from 1 to 2.

A. Effect of Channel Estimation Errors

It was assumed that the receiver knows exactly all the channel coefficients. This is not realistic in all practical situations. Assume that the receiver is optimized for the channel transfer functions $\boldsymbol{C}_i(z)$, but because of channel estimation errors the actual coefficients used when the signal is transmitted is $\hat{\boldsymbol{C}}_i(z)$, where the transfer functions $\boldsymbol{C}_i(z)$ and $\hat{\boldsymbol{C}}_i(z)$ have the same order and size. Let $\hat{\boldsymbol{h}}_i$ contain the $L+1$ scalar channel coefficients corresponding to $\hat{\boldsymbol{C}}_i(z)$. As a measure of the mismatch (MM) between the actual channels $\hat{\boldsymbol{h}}_i$ and the channels used in the optimization $\boldsymbol{h}_i$, $\mathrm{MM}=1/N\sum_{i=0}^{N-1}\|\hat{\boldsymbol{h}}_i-\boldsymbol{h}_i\|^2$ is used. It is assumed that the mismatch is equal for all the N channels. When interpreting the size of MM it is important to remember that $\boldsymbol{h}_i^H\boldsymbol{h}_i=1$. Figure 4 shows the BER versus MM performances of the minimum MSE and minimum BER systems. Since the value of MM depends on the realization of $\hat{\boldsymbol{h}}_i$, Monte Carlo simulations were used. 10000 realizations of the actual channels $\hat{\boldsymbol{h}}_i$ were generated for each each value of MM and then the BER, in (11), was averaged for all these realizations. Figure 4 gives an indication of the sensitivity of the minimum MSE and minimum BER receiver to errors in the channel coefficients.

B. Near-Far Resistance Effect

Let $\boldsymbol{u}_i(n)$ be the noise-free $M\times 1$ vector time-series that is the output of channel $\boldsymbol{C}_i(z)$, see Figure 2. Let P_i be the received signal power from user number i. P_i can be found as: $P_i=E\left[\|\boldsymbol{u}_i(n)\|^2\right]=\mathrm{Tr}\left\{\boldsymbol{C}_{i-}\left[\boldsymbol{I}_2\otimes\boldsymbol{W}_i\boldsymbol{W}_i^H\right]\boldsymbol{C}_{i-}^H\right\}$. Let the channel impulse responses be scaled such that all $P_i=P$ for $i\in\{1,2,\ldots,N-1\}$. The received signal power P_0 from user number 0 can be different than the other received powers. The near-far ratio (NFR) in dB is defined as: $\mathrm{NFR}=10\log_{10}\frac{P_0}{P}$. In Figure 5, the BER_0 versus NFR performance is shown for the DS-CDMA systems having minimum MSE receiver and minimum BER receiver filters. From (11) and (15), it can be deduced that receiver filter number i is chosen such that BER_i is minimized. Since the near-far resistance is measured as BER_0 versus NFR, the proposed system has optimal near-far resistance among linear FIR receivers following the block model in Figure 2.

V. Conclusions

Exact BER were derived for a DS-CDMA system. Based on this expression, a framework was developed for finding linear minimum BER FIR receiver filters. A numerical iterative optimization algorithm was proposed that is able to converge to a locally optimal solution. The proposed receiver filters can be found through a numerical optimization procedure. Numerical examples showed that the proposed minimum BER receivers can perform significantly better than the minimum MSE receivers with the same filter order. Several properties of the minimum FIR BER filters were also identified. Note finally that the results can be also extended to higher order modulations.

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