Spatial Multiplexing by Spatiotemporal Spreading of Multiple Symbol Streams

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Abstract
The use of multiple transmitter and receiver antennas allows to transmit multiple signal streams in parallel and hence to increase communication capacity. To distribute the multiple signal streams over the MIMO channel, linear space-time codes have been shown to be a convenient way to reach high capacity gains with a reasonable complexity. We propose an approach based on spatial spreading and delay diversity, hence spatiotemporal spreading, for channels with delay spread (frequential fading). The approach allows full symbol rate transmission (and hence full capacity) and permits full spatiotemporal diversity. The spatial spreading codes optimized for minimum pairwise error probability (maximum coding gain). Some optimal and suboptimal receiver schemes are discussed also.

Keywords
Spatial multiplexing, MIMO, frequency selective channel, linear precoding, delay diversity, spatial spreading, space-time coding, coding gain, stripping, decision-feedback receiver.

INTRODUCTION
Spatial multiplexing has been introduced independently in a 1994 Stanford University patent by A. Paulraj and by Foschini [1] at Bell Labs. Spatial multiplexing can be viewed as a limiting case of Spatial Division Multiple Access (SDMA) in which the various mobile users are colocated in one single user multi antenna mobile terminal. In that case, the various users are no longer distinguishable on the basis of their (main) direction (DOA) since all antennas are essentially colocated. Nevertheless, if the scattering environment is sufficiently rich, the antenna arrays at TX and RX can see the different DOAs of the multiple paths. One can then imagine transmitting multiple data streams, one stream per path. For this, the set of paths to be used should be resolvable in angle at both TX and RX. Without channel knowledge at the TX, the multiple streams to be transmitted just get mixed over the multiple paths in the matrix channel. They can generally be linearly recovered at the RX if the channel matrix rank equals or exceeds the number of streams. This rank equals the number of paths that are simultaneously resolvable at TX and RX. The assumptions we shall adopt for the proposed approach are no channel knowledge at TX, perfect channel knowledge at RX, frequency-flat channels for the initial part of the paper.

*Eurécom's research is partially supported by its industrial partners: Ascom, Swisscom, Thales Communications, ST Microelectronics, CEGETEL, Motorola, France Télécom, Bouygues Telecom, Hitachi Europe Ltd. and Texas Instruments. The work reported herein was also partially supported by the French RNRT project ERMITAGES.

LINEAR PREFILTERING APPROACH
We shall call here rate the number N_s of symbol sequences (streams, layers) at symbol rate. A general ST coding setup is sketched in Fig. 1. The incoming stream of bits gets transformed to N_s symbol streams through a combination of channel coding, interleaving, symbol mapping and demultiplexing. The result is a vector stream of symbols $\boldsymbol{b}_k$ containing N_s symbols per symbol period. The N_s streams then get

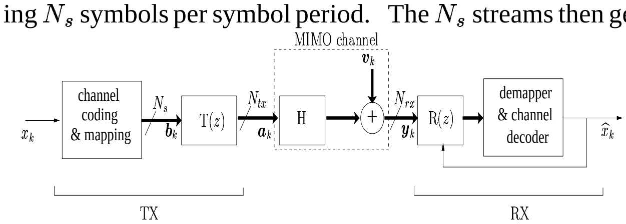

Figure 1. General ST coding setup.

mapped linearly to the N_{tx} transmit antennas and this part of the transmission is called linear ST precoding. The output is a vector stream of symbols $\boldsymbol{a}_k$ containing N_{tx} symbols per symbol period. The linear precoding is spatiotemporal since an element of $\boldsymbol{b}_k$ may appear in multiple components (space) and multiple time instances (time) of $\boldsymbol{a}_k$. The vector sequence $\boldsymbol{a}_k$ gets transmitted over a MIMO channel H with N_{rx} receive antennas, leading to the symbol rate vector received signal $\boldsymbol{y}_k$ after sampling. The linear precoding can be considered to be an inner code, while the nonlinear channel coding etc. can be considered to be an outer code. As the number of streams is a factor in the overall bitrate, we shall call the case $N_s = N_{tx}$ the full rate case, while $N_s = 1$ corresponds to the single rate case. Instead of multiple antennas, more general multiple channels can be considered by oversampling, by using polarization diversity or other EM component variations, by working in beamspace, or by considering in phase and in quadrature (or equivalently complex and complex conjugate) components. In the case of oversampling, some excess bandwidth should be introduced at the transmitter, possibly involving spreading which would then be part of the linear precoding. As we shall see below, channel capacity can be attained by a full rate system without precoding ($\mathrm{T}(z) = I$). In that case, the channel coding has to be fairly intense if we want to exploit all available diversity sources, since it has to spread the information contained in each transmitted bit over space (across TX antennas) and time, see the left part in Fig. 2 and [2]. The goal of introducing the linear precoding is to simplify (possibly going as far as eliminating) the channel coding part [3]. In fact the goal of the linear precoding is to exploit all diversity sources and transform the channel virtually into a non-fading channel so that possible additional channel coding can be taken from the set of non-fading channel codes. In the case of linear dispersion codes [4],[5], transmission is not continuous but packet-wise (block-wise). In that case, a packet of T vector symbols $\boldsymbol{a}_k$ (hence a $N_{tx} \times T$ matrix) gets constructed as a linear combination of fixed matrices in which the combination coefficients are symbols b_k. A particular case is the Alamouti code which is a full diversity single rate code corresponding to block length $T = N_{tx} = 2$, $N_s = 1$. In the first part of this paper we shall focus on continuous transmission in which linear precoding corresponds to MIMO prefiltering. This linear convolutive precoding can be considered as a special case of linear dispersion codes (making abstraction of the packet boundaries) in which the fixed matrices are time-shifted versions of the impulse responses of the columns of $\mathrm{T}(z)$ in Fig. 1. A number of configurations are possible for

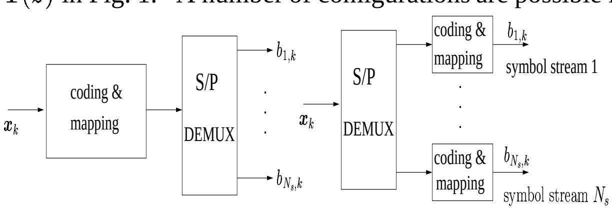

Figure 2. Two channel coding, interleaving, symbol mapping and demultiplexing choices.

the channel coding part (outer code), see Fig. 2. In the global channel coding/mapping case (see left part of Fig. 2), the last operation of the encoding part is spatial demultiplexing (serial-to-parallel (S/P) conversion) (mapping refers to bit interleaving and symbol constellation mapping). At the other extreme, this S/P conversion is the first operation in the case of streamwise channel coding/mapping, see the right part of Fig. 2. An intermediate approach consists of global channel coding followed by S/P conversion and streamwise mapping. Systems without linear precoding require at least streamwise mapping The existing BLAST systems are special cases of such approaches. VBLAST is a full rate system with $\mathrm{T}(z) = \mathrm{I}_{N_{tx}}$ which leads to quite limited diversity in the absence of outer coding. DBLAST (in a simplified version) is a single-rate system with $\mathrm{T}(z) = [1 \;\; z^{-1}, \ldots, z^{-(N_{tx}-1)}]^T$ which leads to full diversity (delay diversity) (on frequency-flat channels). We would like to introduce a prefiltering matrix $\mathrm{T}(z)$ without taking a hit in capacity, while achieving full diversity (in the absence of outer coding). The MIMO prefiltering will allow us to capture all diversity (spatial, and frequential for channels with delay spread) and will provide some coding gain. The optional channel coding then serves to provide additional coding gain and possibly (with interleaving) to capture temporal diversity (Doppler spread) if there is any. In its simplest form, the outer code can consist of global channel coding without interleaving. Some (multi-stream) detection schemes may require stream-wise channel coding though. Finally, though time-invariant filtering may evoke continuous transmission, the prefiltering approach is also immediately applicable to block transmission by replacing convolution by circular convolution (see below).

Capacity

Consider the MIMO AWGN (flat) channel

$$\boldsymbol{y}_k = \mathrm{H}\,\boldsymbol{a}_k + \boldsymbol{v}_k = \mathrm{H}\,\mathrm{T}(q)\,\boldsymbol{b}_k + \boldsymbol{v}_k \tag{1}$$

where the noise power spectral density matrix is $S_{\boldsymbol{vv}}(z) = \sigma_v^2\, I$, $q^{-1}\,\boldsymbol{b}_k = \boldsymbol{b}_{k-1}$. The ergodic capacity when channel knowledge is absent at the TX and perfect at the RX is:

$$\begin{aligned} C(S_{\boldsymbol{aa}}) &= \mathrm{E}_H \frac{1}{2\pi j} \oint \frac{dz}{z} \log_2 \det(I + \frac{1}{\sigma_v^2}\,\mathrm{H}\, S_{\boldsymbol{aa}}(z)\,\mathrm{H}^H) \\ &= \mathrm{E}_H \frac{1}{2\pi j} \oint \frac{dz}{z} \log_2 \det(I + \frac{1}{\sigma_v^2} \mathrm{H}\mathrm{T}(z) S_{\boldsymbol{bb}}(z) \mathrm{T}^\dagger(z)\,\mathrm{H}^H) \\ &= \mathrm{E}_H \frac{1}{2\pi j} \oint \frac{dz}{z} \log_2 \det(I + \rho\,\mathrm{H}\,\mathrm{T}(z)\,\mathrm{T}^\dagger(z)\,\mathrm{H}^H) \end{aligned} \tag{2}$$

where we assume that the outer coding leads to spatially and temporally white symbols: $S_{\boldsymbol{bb}}(z) = \sigma_b^2\, I$, and $\rho = \frac{\sigma_b^2}{\sigma_v^2} = \frac{SNR}{N_{tx}}$. The expectation E_H is here w.r.t. the distribution of the channel. As in [6], we assume the channel entries $\mathrm{H}_{i,j}$ to be mutually independent zero mean complex Gaussian variables with unit variance (Rayleigh flat fading MIMO channel model). As stated in [7], to avoid capacity loss the prefilter $\mathrm{T}(z)$ is required to be paraunitary ($bfT(z)\mathrm{T}^\dagger(z) =$ I) (hence full stream TX is required). Motivated by the consideration of diversity also (see below), we propose to use the following paraunitary prefilter

$$\mathrm{T}(z) = \mathrm{D}(z)\;Q,\;\; \mathrm{D}(z) = diag\{1, z^{-1}, \ldots, z^{-(N_{tx}-1)}\} \tag{3}$$

where $\mathrm{D}(z)$ introduces delay diversity and Q is a (constant) unitary matrix with equal magnitude elements, $|Q_{ij}| = \frac{1}{\sqrt{N_{tx}}}$, that performs spatial spreading (columns are spatial spreading codes). Note that for a channel with a delay spread of L symbol periods, the prefilter can be immediately adapted by replacing the elementary delay z^{-1} by z^{-L}. For the propagation channel $\mathrm{H}(z)$ (with columns $\mathrm{H}_{:,i}(z)$) combined with the prefilter $\mathrm{T}(z)$ in (3), symbol stream n ($b_{n,k}$) passes through the equivalent SIMO channel

$$\sum_{i=1}^{N_{tx}} z^{-(i-1)L}\;\mathrm{H}_{:,i}(z)\;Q_{i,n} \tag{4}$$

which now has extended memory due to the delay diversity introduced by $\mathrm{D}(z)$. It is important that the different columns $\mathrm{H}_{:,i}$ of the channel matrix get spread out in time to get full diversity (otherwise the streams just pass through a linear combination of the columns, which would offer the same limited diversity as in VBLAST). The delay diversity only becomes effective by the introduction of the mixing/rotation matrix Q, which has equal magnitude elements for uniform diversity source exploitation.

Matched Filter Bound and Diversity

The Matched Filter Bound (MFB) is the maximum attainable SNR for symbol-wise ML detection, when the interference from all other symbols has been removed. Hence the multistream MFB equals the MFB for a given stream. For VBLAST ($\mathrm{T}(z)=\mathrm{I}$), the MFB for stream n is

$$\mathrm{MFB}_n = \rho \|\mathrm{H}_{:,n}\|_2^2 \tag{5}$$

hence, diversity is limited to N_{rx}. For the proposed $\mathrm{T}(z) = \mathrm{D}(z)\,Q$ on the other hand, stream n has MFB

$$\mathrm{MFB}_n = \rho \frac{1}{N_{tx}}\, \|\mathrm{H}\|_F^2 \tag{6}$$

hence this $\mathrm{T}(z)$ provides the same full diversity $N_{tx}N_{rx}$ for all streams. Larger diversity order leads to larger outage capacity.

Pairwise Error Probability P_e (Flat Channel Case)

The received signal is:

$$\boldsymbol{y}_k = \mathrm{H}\;\mathrm{D}(q)\;Q\;\boldsymbol{b}_k + \boldsymbol{v}_k = \mathrm{H}\;\mathrm{D}(q)\;\boldsymbol{c}_k + \boldsymbol{v}_k \tag{7}$$

where $\boldsymbol{c}_k = Q\,\boldsymbol{b}_k = [c_1(k)\,c_2(k)\,\ldots\,c_{N_{tx}}(k)]^T$. We consider now the transmission of the coded symbols over a duration of T symbol periods, $T >> N_{tx}$ so that the loss in rate introduced by the insertion of a guard interval of length $N_{tx}-1$ is negligible. The accumulated received signal is then:

$$\mathrm{Y} = \mathrm{H}\;\mathrm{C} + \mathrm{V} \tag{8}$$

where Y and V are $N_{rx} \times T$ and C is $N_{tx} \times T$. The structure of C is:

$$\mathrm{C}=\begin{bmatrix} c_1(0) & c_1(1) \ldots c_1(T-N_{tx}) & 0 & \ldots & 0 \\ 0 & \ddots\ \ddots & \ldots\ c_2(T-N_{tx}) & \ddots & \vdots \\ \vdots & \ddots\ \ddots & \ddots & \ldots & \ddots & 0 \\ 0 & \ldots\ 0 & c_{N_{tx}}(0) & c_{N_{tx}}(1) \ldots c_{N_{tx}}(T-N_{tx}) \end{bmatrix} \tag{9}$$

Over a Rayleigh flat fading i.i.d. MIMO channel, the probability of deciding erroneously C' for transmitted C is upper bounded by (see [3]):

$$\mathrm{P}(\mathrm{C}\to\mathrm{C}') \le \prod_{i=1}^{\boldsymbol{r}} (1+\frac{\rho}{4}\lambda_i)^{-N_{rx}} \tag{10}$$

Where for high SNR this becomes:

$$\mathrm{P}(\mathrm{C}\to\mathrm{C}') \le (\prod_{i=1}^{\boldsymbol{r}} \lambda_i)^{-N_{rx}} (\frac{\rho}{4})^{-N_{rx}\,\boldsymbol{r}} \tag{11}$$

where $\boldsymbol{r}$ and λ_i are rank and eigenvalues of $\frac{1}{\sigma_b^2}(\mathrm{C}-\mathrm{C}')(\mathrm{C}-\mathrm{C}')^H$. Introduce $\boldsymbol{e}(k) = \frac{1}{\sigma_b}(\boldsymbol{c}_k - \boldsymbol{c}'_k)$, then:

$$\mathrm{C}-\mathrm{C}' = \sigma_b \begin{bmatrix} e_1(0) & e_1(1) & \ldots & \ldots & \ldots & \ldots \\ 0 & \ddots & \ddots & \ldots & \ldots & \ldots \\ \vdots & \ddots & \ddots & \ddots & \ldots & \ldots \\ 0 & \ldots & 0 & e_{N_{tx}}(0) & e_{N_{tx}}(1) & \ldots \end{bmatrix} \tag{12}$$

Let i be the time index of the first error:

$$\mathrm{C}-\mathrm{C}' = \sigma_b \begin{bmatrix} 0 & \ldots & 0 & e_1(i) & \ldots & \ldots & \ldots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ldots & \ldots \\ 0 & \ldots & \ldots & \ldots & 0 & e_{N_{tx}}(i) & \ldots \end{bmatrix}. \tag{13}$$

$$\text{Assume} \quad \prod_{n=1}^{N_{tx}} e_n(i) \neq 0 \tag{14}$$

then the upper bound on the pairwise error probability becomes maximized for a single error event i:

$$\mathrm{P}(\mathrm{C}\to\mathrm{C}') \le (\prod_{n=1}^{N_{tx}} |e_n(i)|^2)^{-N_{rx}} \cdot (\frac{\rho}{4})^{-N_{rx}\,N_{tx}} \quad . \tag{15}$$

Hence, full diversity $N_{rx}\,N_{tx}$ is guaranteed, and the coding gain is: $\min\limits_{\boldsymbol{e}(i)\neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2$. The condition (14) is well known in the design of lattice constellations (see [8], [9]), a field based on the theory of numbers. A solution that satisfies our criteria of unitary matrix and equal magnitude components of Q, is the Vandermonde matrix:

$$Q^s = \frac{1}{\sqrt{N_{tx}}} \begin{bmatrix} 1 & \theta_1 & \ldots & \theta_1{}^{N_{tx}-1} \\ 1 & \theta_2 & \ldots & \theta_2{}^{N_{tx}-1} \\ \vdots & \vdots & & \vdots \\ 1 & \theta_{N_{tx}} & \ldots & \theta_{N_{tx}}{}^{N_{tx}-1} \end{bmatrix} \tag{16}$$

where the θ_i are the roots of $\theta^{N_{tx}} - j = 0$, $j=\sqrt{-1}$.
It was shown in [7] that when $N_{tx} = 2^{n_t}$ ($n_t \in \mathrm{N}$), and for a finite QAM constellation with $(2M)^2$ points, then Q^s maximizes the coding gain among all matrix Q with normalized columns, and achieves: $\min\limits_{\boldsymbol{e}(i)\neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2 = \left(\frac{d^2}{N_{tx}\sigma_b^2}\right)^{N_{tx}} = \left(\frac{6}{N_{tx}(4M^2-1)}\right)^{N_{tx}}$, where d is the minimum distance between two points in the constellation.

From Continuous TX to Block Transmission

The size of the block is T symbol periods. Even if $T \gg N_{tx}$ the insertion of a guard interval leads to a non-zero $\frac{N_{tx}-1}{T}$ fraction loss in the original rate. A way to avoid this is to use circular convolution (or wrapping). The inconvenience of this though is that the codeword difference matrix $\mathrm{C}-\mathrm{C}'$ is no longer triangular; the study of the coding gain hence gets more involved. For $j \ge i$, $\boldsymbol{e}(i) \neq 0$ and $\boldsymbol{e}(j) \neq 0$ are two successive errors if they verify one of the following two conditions:

- $\forall k \in 1,\ldots,j-i-1,\ \boldsymbol{e}(i+k) = 0$
- or $\forall k \in 1,\ldots,T-j+i-1,\ \boldsymbol{e}((j+k)mod T) = 0$

The codeword difference matrix is:

$$\mathrm{C}-\mathrm{C}' = \sigma_b \cdot \begin{bmatrix} e_1(0) & e_1(1) & \cdots & \cdots & \cdots & e_1(T-2) & e_1(T-1) \\ e_2(T-1) & & & & & & e_2(T-2) \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ e_{N_{tx}}(T-N_{tx}+1)\ldots & e_{N_{tx}}(T-1) & e_{N_{tx}}(0) & \cdots & \cdots & & e_{N_{tx}}(T-N_{tx}) \end{bmatrix}$$

Let $S_e = \{(i,j)\,|j \geq i\,,\,\boldsymbol{e}(i),\,\boldsymbol{e}(j)$ are 2 successive errors$\}$ be the field of successive errors.

If there exists $(i_0, j_0) \in S_e$ with $j_0 - i_0 \geq N_{tx}$, then in the same way as has been argued in the previous section we can bound the error probability by the single error probability of $e(i_0)$ (equivalently of $e(j_0)$); this yields the same result for diversity and coding gain as stated before. Now, for the event when there are no successive errors separated by more than $N_{tx}-1$, there are at least $\frac{T}{N_{tx}-1}$ non zero errors $\boldsymbol{e}(i)$. >From (10) a bound for the error probability of this event is given by:

$$\begin{aligned} &\mathrm{P}(\mathrm{C}\to\mathrm{C}') \leq \textstyle\prod_{i=1}^{\boldsymbol{r}}(1+\frac{\rho}{4}\lambda_i)^{-N_{rx}} \leq (1+\frac{\rho}{4}\sum_{i=1}^{\boldsymbol{r}}\lambda_i)^{-N_{rx}} \\ &= (1+\tfrac{\rho}{4}\tfrac{1}{\sigma_b^2}||\mathrm{C}-\mathrm{C}'||_F^2)^{-N_{rx}} = (1+\tfrac{\rho}{4}\textstyle\sum_{i=0}^{T-1}||\boldsymbol{e}(i)||_2^2)^{-N_{rx}} \\ &\leq (1+\tfrac{\rho}{4}\tfrac{T}{N_{tx}-1}\min_{\boldsymbol{e}(i)\neq 0}||\boldsymbol{e}(i)||_2^2)^{-N_{rx}} \\ &= (1+\tfrac{\rho}{4}\tfrac{T}{N_{tx}-1}\tfrac{6}{4M^2-1})^{-N_{rx}} \qquad (17) \end{aligned}$$

The probability of such an error event is less then the upper bound given in (11), and hence conserves this bound when $SNR \leq \frac{2}{3}N_{tx}^2(4M^2-1)\,T^{\frac{1}{N_{tx}-1}}$. This is contains the SNR range of interest for most applications.

Frequency Selective Channel Case

The multipath channel now has a finite delay spread of L symbol periods: $\mathrm{H}(z) = \sum_{i=0}^{L-1}\mathrm{H}_i z^{-i}$. The MIMO channel coefficients $\mathrm{H}_i, i = 1, \ldots, L$ are considered to be independent, and the elements of H_i are assumed to be iid Gaussian variables with mean 0 and variance σ_i^2. We propose as precoding matrix $\mathrm{T}(z) = \mathrm{D}(z^L)\,Q$. The received signal is then:

$$\mathrm{Y} = \mathrm{G}\;\mathrm{S}\;\tau(\mathrm{C}) + \mathrm{V} \qquad (18)$$

where $\mathrm{S} = diag\{\sigma_0 I_{N_{tx}}, \ldots, \sigma_{L-1}I_{N_{tx}}\}, \mathrm{G} = [\mathrm{H}_0, \mathrm{H}_1, \ldots, \mathrm{H}_{L-1}]\mathrm{S}^{-1}$ and $\tau(\mathrm{C})$ is a block Toeplitz matrix with C as the first block row, where C is:

$$\mathrm{C} = \begin{bmatrix} c_1(0) & c_1(1) & \cdots & \cdots & \cdots & \cdots \\ & 0_{1\times L} & c_2(0) & c_2(1) & \cdots & \cdots \\ & \vdots & \ddots & \ddots & \cdots & \cdots \\ & 0_{1\times(N_{tx}-1)L} & & & c_{N_{tx}}(0) & \cdots \end{bmatrix} \qquad (19)$$

G has iid normalized Gaussian elements. The upper bound for the pairwise error probability given in (11) is still valid, where $\boldsymbol{r}$ and λ_i are now the rank and eigenvalues of $\frac{1}{\sigma_b^2}\mathrm{S}(\tau(\mathrm{C}) - \tau(\mathrm{C}'))(\tau(\mathrm{C}) - \tau(\mathrm{C}'))^H\mathrm{S}^H$.

Let's define the permutation matrix $\mathcal{P}$ of size $N_{tx}L \times N_{tx}L$ such that: $\mathcal{P}\boldsymbol{u}_k = \boldsymbol{u}_{p(k)}$ where u_k is the $N_{tx}L \times 1$ vector with 1 in the k^{th} position and zeros elsewhere, $p(k) = (k-1\,mod\,N_{tx})\,L + (k\,div\,N_{tx}) + 1$. Permuting the rows of $\tau(\mathrm{C}) - \tau(\mathrm{C}')$ gives:

$$\mathcal{P}\,(\tau(\mathrm{C}) - \tau(\mathrm{C}')) = \sigma_b \begin{bmatrix} \mathrm{E}_1 & \\ 0_{L\times L} & \mathrm{E}_2 \\ 0_{L\times 2L} & \mathrm{E}_3 \\ \vdots & \vdots \\ 0_{L\times(N_{tx}-1)L} & \mathrm{E}_{N_{tx}} \end{bmatrix} \qquad (20)$$

where

$$\mathrm{E}_k = \begin{bmatrix} e_k(0) & e_k(1) & \cdots & \cdots & \cdots & \cdots \\ 0 & \ddots & \ddots & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \ddots & \cdots & \cdots \\ & 0_{1\times(L-1)} & & e_k(0) & e_k(1) & \cdots \end{bmatrix} \qquad (21)$$

As stated for the flat channel case, the pairwise error probability $\mathrm{P}(\mathrm{C}\to\mathrm{C}')$ is upper bounded by:

$$\left(\prod_{k=0}^{L-1}\sigma_k^2\right)^{-N_{rx}\,N_{tx}} \left(\prod_{n=1}^{N_{tx}}|e_n(i)|^2\right)^{-N_{rx}\,L} \left(\frac{\rho}{4}\right)^{-N_{rx}\,N_{tx}\,L} \qquad (22)$$

where i is the time index of the first error. >From the exponent of ρ we can conclude that the proposed scheme exploits the full diversity (degree $N_{rx}\,N_{tx}\,L$). The optimality result of the proposed Q given in the previous section in the sense that it maximizes the coding gain $\min_{\boldsymbol{e}(i)\neq 0}\prod_{n=1}^{N_{tx}}|e_n(i)|^2 \sim N^{-N}$ (N = size of Q) is also valid for the frequency selective channel. An alternative approach to handling the frequency selective channel case uses OFDM and involves a Q of size NL. The increase in size of Q leads to a substantial decrease in coding gain though. For the case when we use circular convolution with a block of size T, the same analysis holds as for the case of a flat channel, with the upper bound of the error probability valid for $SNR \leq \frac{2}{3}LN_{tx}^2(4M^2-1)\,T^{\frac{1}{N_{tx}L-1}}$, where we have assumed a flat power delay profile for the channel: $\sigma_i^2 = \frac{1}{L}, i = 1, \ldots, L$.

ML RECEPTION

In principle, we can perform Maximum Likelihood reception since the delay diversity transforms the (flat) channel into a channel with finite memory. However, the number of states would be the product of the constellation sizes of the N_{tx} streams to the power $LN_{tx}-1$. Hence, if all the streams have the same constellation size $|\mathcal{A}|$, the number of states would be $|\mathcal{A}|^{N_{tx}(LN_{tx}-1)}$, which will be much too large in typical applications. Suboptimal ML reception can be performed in the form of sphere decoding [10]. The complexity of this can still be too large though. Alternatively, PIC and turbo RXs can be used as approximations to ML reception. Another suboptimal receiver structure will be considered in the next section.

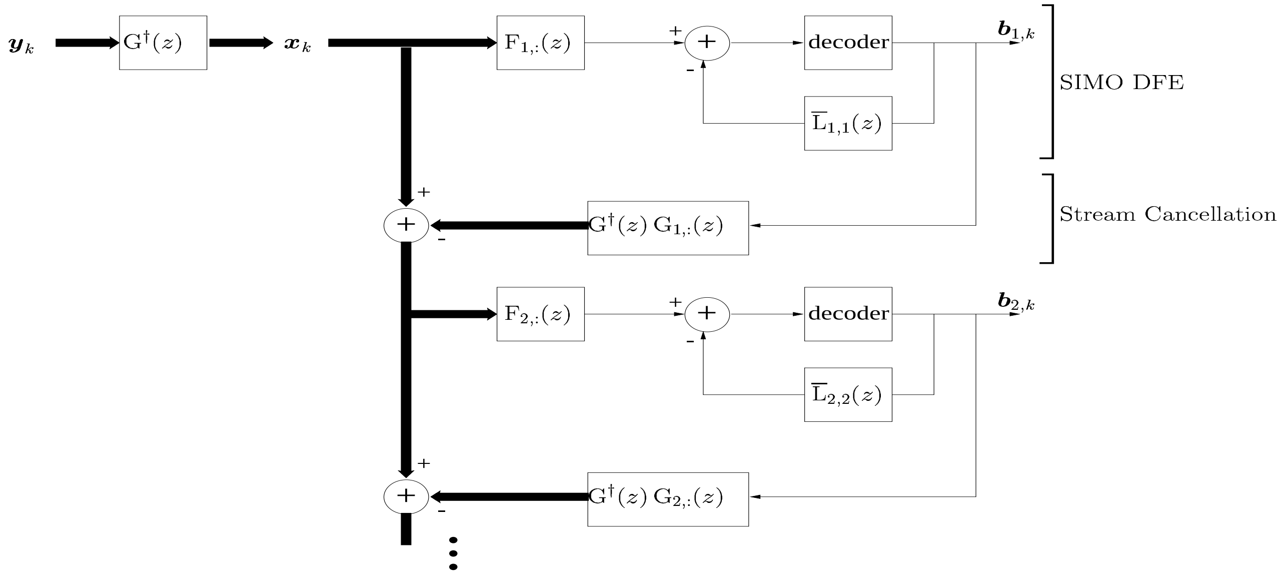

Figure 3. Triangular MIMO DFE Receiver.

STRIPPING MIMO DFE (SIC) RECEPTION

Let $G(z) = H(z)\,T(z)$ be the cascade transfer function of channel and precoding. The matched filter RX is

$$\begin{aligned} \boldsymbol{x}_k = G^\dagger(q)\,\boldsymbol{y}_k &= G^\dagger(q)\,G(q)\,\boldsymbol{b}_k + G^\dagger(q)\,\boldsymbol{v}_k \\ &= R(q)\,\boldsymbol{b}_k + G^\dagger(q)\,\boldsymbol{v}_k \end{aligned} \tag{23}$$

where $R(z) = G^\dagger(z)\,G(z)$, and the psdf of $G^\dagger(q)\,\boldsymbol{v}_k$ is $\sigma_v^2\,R(z)$. The MIMO DFE RX is then:

$$\widehat{b}_k = -\underbrace{\overline{L}(q)}_{\text{feedback}}\,b_k + \underbrace{F(q)}_{\text{feedforward}}\,x_k \tag{24}$$

where feedback $\overline{L}(z)$ is strictly "causal" (causal is here first between users and then in time: $L(z) = I + \overline{L}(z)$ is lower triangular with causal diagonal). Fig. 3 illustrates that this MIMO DFE corresponds to SIMO DFE's per stream plus cancellation of each detected stream from the RX signal (or MF output) before detection of the next stream. This scheme is hence the extension of the VBLAST "nulling (in the ZF case) and canceling" RX to the spatiotemporal case. Two design criteria for feedforward and feedback filters are possible: (MMSE) ZF and MMSE, see [7], where we indicated that triangular MIMO feedback structures allow to incorporate channel decoding before cancellation, which leads to the stripping approach of Verdu & Müller or Varanasi & Guess. Simplified RXs can be obtained by the use of a (noise) predictive DFE which allows to approximate the (LMMSE) forward filter via polynomial expansion (filtering with $R(z)$) and to reduce the order of the feedback filter (predictor) to a desired complexity level.

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